Search for the Rare Decay $K_L \to \pi^0\pi^0\gamma$

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The KTeV E799 experiment has conducted a search for the rare decay $K_L \to \pi^0\pi^0\gamma$ via the topology $K_L \to \pi^0\pi^0_D\gamma$ (where $\pi^0_D \to \gamma e^+e^-$). Due to Bose statistics of the π^0 pair and the real nature of the photon, the $K_L \to \pi^0\pi^0\gamma$ decay is restricted to proceed at lowest order by the CP conserving direct emission (DE) of a E2 electric quadrupole photon. The rate of this decay is interesting theoretically since chiral perturbation theory predicts that this process vanishes at level $O(p^4)$. Therefore, this mode probes chiral perturbation theory at $O(p^6)$. In this paper we report a determination of an upper limit of 2.43×10^{-7} (90% CL) for $K_L \to \pi^0\pi^0\gamma$. This is approximately a factor of 20 lower than previous results.

While calculations of branching ratios for kaon decay modes such as $K_L \to \gamma\gamma$ have shown good agreement [1] with chiral perturbation theory ChPT calculations to order $O(p^4)$, ChPT calculation to order $O(p^6)$ have been difficult to check due to the presence of the large lower order terms. Since the $K_L \to \pi^0\pi^0\gamma$ mode is forbidden to order $O(p^4)$ in (ChPT) [2], this mode presents an opportunity to check ChPT at higher orders. Therefore an experimental measurement of this mode's branching ratio is of interest. At present, a chiral dimensional analysis [3] for the $K_L \to \pi^0\pi^0\gamma$ mode results in a branching ratio of 7×10^{-11}. Because of the Bose statistics of the $\pi^0\pi^0$ pair and the presence of a real direct emission γ, the $K_L \to \pi^0\pi^0\gamma$ decay proceeds at lowest order via the CP conserving direct emission of a electric quadrupole (E2) photon. This causes a large suppression of this mode relative to the associated $K_L \to \pi^+\pi^-\gamma$ mode. Accordingly, observation of $K_L \to \pi^0\pi^0\gamma$ would give a measurement of E2 quadrupole emission which is difficult to extract from the E1 and M1 dominated $K_L \to \pi^+\pi^-\gamma$ mode. The M1 amplitude determined from $K_L \to \pi^+\pi^-\gamma$ decay together with a plausible estimate for E2 amplitude [4],

given by

$$A(\pi^0\pi^0\gamma) = \frac{g_{E2}}{M_K^4}\frac{(p_1-p_2)\cdot k}{\Lambda^2}\times\epsilon\cdot(p_1 k\cdot p_2 - p_2 k\cdot p_1), \quad (1)$$

can be used to obtain an estimate of the branching ratio for $K_L \to \pi^0\pi^0\gamma$. Here p_1 and p_2 are the momenta of the pions and k and ϵ are the momentum and polarization of the photon respectively. g_{E2} is the coupling of the E2 amplitude. Λ^{-1}, a mass parameter that measures the extent of the region over which the interaction takes place, is assumed to be of order of the ρ meson mass. The CP violating direct emission of a M2 magnetic quadrupole photon is much smaller than the E2 transition and can be ignored. According to Ref. [4], the decay rate for $K_L \to \pi^0\pi^0\gamma$ can be estimated by comparing it to the M1 direct emission (DE) amplitude of the $K_L \to \pi^+\pi^-\gamma$ decay

$$A(\pi^+\pi^-\gamma) = \frac{g_{M1}}{M_K^4}\epsilon_{\mu\nu\rho\sigma}\epsilon^\mu k^\nu p_+^\rho p_-^\sigma \quad (2)$$

Employing this amplitude, the ratio

$$\frac{\Gamma(K_L \to \pi^0\pi^0\gamma)}{\Gamma(K_L \to \pi^+\pi^-\gamma)_{DE}} = \frac{1}{2}\left(\frac{g_{E2}}{g_{M1}}\right)^2\left(\frac{M_K^2}{\Lambda^2}\right)^2(4\times 10^{-3}). \quad (3)$$

can be formed. Using the measured M1 direct emission rate for $K_L \to \pi^+\pi^-\gamma$ of $(14.2 \pm 0.28) \times 10^{-3}$ normalized to the $K_L \to \pi^+\pi^-$ from Ref. [5] and a $K_L \to \pi^+\pi^-$ branching ratio of $(2.090 \pm 0.025) \times 10^{-3}$ from the PDG [6], a branching ratio for $K_L \to \pi^+\pi^-\gamma$ of 2.96×10^{-5} ($E_\gamma \geq 20 \text{MeV}$) is obtained. Using Eq. 3 above, this gives an estimated branching ratio of 1.08×10^{-8} for $K_L \to \pi^0\pi^0\gamma$ (under the assumption that g_{E2} has similar magnitude to g_{M1}).

The KTeV collaboration previously reported an upper limit of of 5.4×10^{-9} (90% CL) [7] for the related decay $K_L \to \pi^0\pi^0 e^+e^-$ in which the direct emission E2 photon was virtual. However, we point out that this decay, in contrast to the $K_L \to \pi^0\pi^0\gamma$ decay where the photon is real, can proceed via an additional process forbidden for a real photon. The virtual photon is emitted in a J=0 state from the K_L allowing a transition of the K_L to a K_S followed by the CP allowed decay $K_S \to \pi^+\pi^-$. This is the so-called charge radius amplitude [8]. The rates for these two decays would have similar branching ratios except for the the charge radius amplitude.

The previous best upper limit of 5×10^{-6} (90% CL) for the $K_L \to \pi^0\pi^0\gamma$ decay obtained by the NA31 experiment [9] was achieved by searching for this mode in events with five photons in the final state. Due to the particular configuration of triggers and prescale factors in the KTeV experiment, the most sensitive method was to require that one of the neutral pions undergo Dalitz decay $\pi^0 \to e^+e^-\gamma$.

The search for the $K_L \to \pi^0\pi^0_D\gamma$ mode was performed using the 1997 and 1999 runs of KTeV E799 II at Fermi National Accelerator Laboratory. The KTeV E799 experiment used two almost parallel K_L beam lines generated by interactions of 800 GeV/c protons in a BeO target some 90 meters upstream of the KTeV spectrometer. The KTeV spectrometer consisted of a 70 meter evacuated decay tube followed by, in sequence, two stations of drift chambers, a large aperture dipole magnet, two more stations of drift chambers, a multi-plane transition radiation detector, a 3100 element CsI electromagnetic calorimeter and a muon shield.

The $K_L \to \pi^0\pi^0_D\gamma$ final state consists of four photons plus an e^+e^- pair. The KTeV analysis required six electromagnetic showers ($E \geq 0.6$ GeV) in the CsI calorimeter, two of which were associated with two reconstructed charged tracks. The two charged tracks were required to be consistent with the two track trigger, form a good charged track vertex, have opposite charge, and have $0.95 \leq E/p \leq 1.05$ (where E is the energy of the associated shower in the CsI calorimeter and p is the momentum determined from magnetic bending).

Events passing these cuts were required to satisfy several additional cuts to select $K_L \to \pi^0\pi^0\gamma$ and to reject the major background due to $K_L \to \pi^0\pi^0\pi^0_D$ decays. These events can contribute to the background if one of the photons was not detected or two of the photons re-

constructed as single photon.

Additional cuts were performed including a vertex cut determined by cycling over all $\gamma\gamma$ and $e^+e^-\gamma$ combinations in a given event for all possible vertex positions to determine the vertex where the $M_{e^+e^-\gamma}$ and $M_{\gamma\gamma}$ were closest to the π^0 mass. The vertex thus obtained was required to be between 95 and 150 meters from the BeO target in the KTeV decay volume and to have a good vertex χ^2. Another cut was made on the variable $(P_L^2)_{\pi^0}$ defined as the longitudinal momentum squared of the π^0 in the frame in which the momentum of the $\pi^0\pi^0_D$ pair is totally transverse. If the event is a $K_L \to \pi^0\pi^0\pi^0_D$ decay, $(P_L^2)_{\pi^0}$ is greater than zero. If the event is a $K_L \to \pi^0\pi^0_D\gamma$ decay, $(P_L^2)_{\pi^0}$ is less than zero (modulo resolution). Accordingly $(P_L^2)_{\pi^0}$ was required ≤ -0.005 GeV^2/c^2 in order to select $K_L \to \pi^0\pi^0_D\gamma$ decays and reject $K_L \to \pi^0\pi^0\pi^0_D$.

Monte Carlos were developed for the signal $K_L \to \pi^0\pi^0_D\gamma$ mode incorporating the model of Ref [4] as well as for the various backgrounds, and the normalization mode incorporating all known features of the E799 spectrometer and beam. The various K_L modes were generated using experimentally determined decay parameters. Trigger and analysis cuts applied to the Monte Carlos were the same as those applied to the data. Background and normalization mode Monte Carlos were generated to produce Monte Carlo samples several times the 1997 and 1999 run K_L fluxes.

The signal Monte Carlo indicated that the angle θ between the direct emission photon and one of the two π^0's in the $\pi^0\pi^0$ center of mass is peaked for the signal mode at $\cos\theta = \pm 0.6$ and flat for $K_L \to \pi^0\pi^0\pi^0_D$. Therefore a final cut to select $K_L \to \pi^0\pi^0_D\gamma$ was made requiring events to have $0.25 \leq |\cos\theta| \leq 0.9$ to improve signal to background. Several other backgrounds were investigated to see if there was any contribution after the cuts to eliminate $K_L \to \pi^0\pi^0_D\gamma$. These included backgrounds due to $K_L \to \pi^0\pi^0_D\pi^0_D$ where a Dalitz pair is undetected, $K_L \to \pi^0\pi^0\pi^0$ followed by the decay of a π^0 into an e^+e^- pair, $K_L \to \pi^0\pi^0\pi^0$ followed by the decay of a π^0 into $e^+e^-e^+e^-$ with one of the e^+e^- pairs undetected, $K_L \to \pi^0 e^+e^-\gamma$ decays with accidental photons, and $K_L \to \pi^0\pi^0_D$ plus accidental photons. In no case did any of these modes contribute significantly to the background. The agreement of the Monte Carlo simulation of background remaining after all cuts with the 1997 and 1999 data outside the signal region (defined below) is shown in Fig. 1.

The signal regions for the 1997 and 1999 data were based on the $M_{ee\gamma\gamma\gamma\gamma}$ and $p_t^2(ee\gamma\gamma\gamma\gamma)$ resolutions calculated using a signal mode Monte Carlo. For the 1997 data, the signal region was chosen to be rectangular with 0.494 GeV/c$^2 \leq$ M ≤ 0.501 GeV/c^2 and $p_t^2 \leq 0.00015$ (GeV/c)^2. In the later analysis of the 1999 data, it was decided to use the 90% CL contour based on the resolutions for $p_t^2(ee\gamma\gamma\gamma\gamma)$ and $M_{ee\gamma\gamma\gamma\gamma}$. Since the signal box
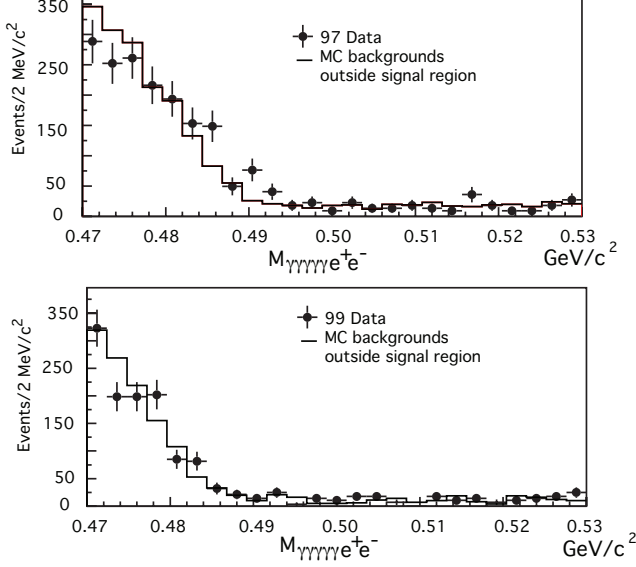
FIG. 1: Comparison of data and MC backgrounds outside of the signal box for the 1997 and 1999 data

had already been opened for the 1997 data (and no events were observed as discussed below), it was decided to stay with a rectangular signal region for the 1997 data.

As shown in Fig. 2a, no data events were observed in this region when the box was opened after the blind analysis was completed. From the $K_L \to \pi^0\pi^0\pi^0$ background Monte Carlo, we expected 0.83 ± 0.41 background events in the 1997 run. To the contrary, Fig. 2b) shows that one data event was found in the 1999 signal region while the background $K_L \to \pi^0\pi^0\pi^0_D$ MC yielded no events in the signal region for four equivalent K_L fluxes.

Since the $K_L \to 3\pi^0_D$ branching ratio was well measured and since events where a photon was lost down the beam hole had the same topology as the signal and were copious, these events were used for the normalization for the experiment. The criteria to select these normalization mode events included the same topology criteria, the vertex position cut, and the π^0 mass cuts as the signal mode. Cuts that were special to the selection of the normalization mode included a requirement that $(P_L^2)_{\pi^0} \geq 0.0$. In addition, to insure that the event was really a $K_L \to 3\pi^0_D$ decay with a photon going down the beam hole, the CsI beam hole photon was reconstructed by assuming a direction of the photon toward the center of the beam hole. Using the additional assumptions of kaon mass and net zero p_t^2 for the event, two solutions for the energy of the photon lost down the beam hole could be calculated.

If one of the solutions for the lost photon resulted in a $\gamma\gamma$ mass within 0.05 GeV/c^2 of the π^0 mass when combined with the unpaired "direct" emission photon (the
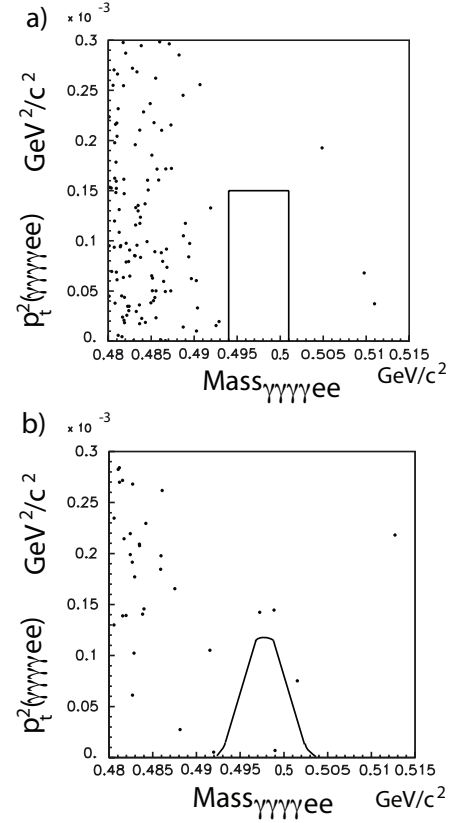

FIG. 2: a) $M_{ee\gamma\gamma\gamma\gamma}$ vs. $p_t^2(ee\gamma\gamma\gamma\gamma)$ for the 1997 KTeV data; b) $M_{ee\gamma\gamma\gamma\gamma}$ vs. $p_t^2(ee\gamma\gamma\gamma\gamma)$ for the 1999 KTeV data

other three photons and the e^+e^- in the event having been paired to form π^0's), the event was designated as a normalization event. If both solutions satisfied this criteria, the one closest to the π^0 mass was used. Normalization mode yields of 7725 and 9024 $K_L \to 3\pi^0_D$ were obtained for the 1997 and 1999 data using these criteria.

We can avoid the extra error due to the uncertainty in the Dalitz decay by noting that the single event sensitivities (SES) for $K_L \to \pi^0\pi^0_D\gamma$ are given by

$$SES(\pi^0\pi^0_D\gamma)_{97,99} = \frac{1}{A(\pi^0\pi^0_D\gamma)_{97,99}N(K_L)_{97,99}} \quad (4)$$

where $A(\pi^0\pi^0_D\gamma)_{97,99}$ are the acceptances for the $K_L \to \pi^0\pi^0_D\gamma$ Dalitz decay mode and $N(K_L)_{97,99}$ are the K_L fluxes for the 1997 and 1999 modes. Since the fluxes are given by

$$N(K_L)_{97,99} = \frac{N(3\pi^0_D)^{norm}_{97,99}}{A(3\pi^0_D)^{norm}_{97,99}BR(K_L \to 3\pi^0_D)} \quad (5)$$

the SES for $K_L \to \pi^0\pi^0\gamma \to \gamma\gamma\gamma\gamma\gamma$ can be written as

$$SES_{97,99} = \frac{3BR(K_L \to 3\pi^0)BR(\pi^0 \to \gamma\gamma)A(3\pi^0_D)^{norm}_{97,99}}{2A(\pi^0\pi^0_D\gamma)_{97,99}N(3\pi^0_D)^{norm}_{97,99}}$$

$$(6)$$

Error on the $3\pi^0$ branching ratio	0.61%
Error on the $\pi^0 \to \gamma\gamma$ branching ratio	3.24%
Normalization MC/data disagreements	3.57%
Signal MC/normalization mode MC disagreements	5.35%
Total Systematic Error	7.23%

TABLE I: % uncertainties for the $K_L \to \pi^0\pi^0\gamma$ SES

The factor of 2 and 3 arise from the number of π^0's that can produce a Dalitz pair in the signal and normalization modes. Note that the Dalitz decay factor cancels out in the SES for $K_L \to \pi^0\pi^0\gamma$.

From the $K_L \to \pi^0\pi^0_D\gamma$ and $K_L \to \pi^0\pi^0\pi^0_D$ Monte Carlos, the signal and normalization acceptances were determined to be 0.101% and 4.21×10^{-6} for the 1997 run and 0.085% and 3.26×10^{-6} for the 1999 runs respectively. Using Eq. 6 and the measured 19.56±0.12% branching ratio for $K_L \to 3\pi^0$ [10], the 1997 and 1999 SES's were determined to be 1.56×10^{-7} and 1.25×10^{-7} respectively. In addition, K_L fluxes of 2.68×10^{11} and 3.99×10^{11} were obtained for the 1997 and 1999 runs. The SES's for 97/99 were combined using

$$\frac{1}{SES_{combined}} = \frac{1}{SES_{97}} + \frac{1}{SES_{99}} \qquad (7)$$

resulting in a combined SES of 6.93×10^{-8}.

Systematic errors for the upper limit could arise from several sources. Among them was the error in the branching ratio for $K_L \to 3\pi^0$. There were also disagreements between the distributions of the $K_L \to 3\pi^0_D$ normalization mode data and Monte Carlo. The effect of these disagreements on the SES were studied by adjusting the Monte Carlos to eliminate the disagreements and seeing what changes in the SES took place. There could also be disagreements between the $K_L \to \pi^0\pi^0_D\gamma$ data and its Monte Carlo. A disagreement in signal mode data and Monte Carlo could not be checked directly since no signal is observed. However, from an inspection of the calculation of the SES, it can be seen that the SES is proportional to $(N_{signal}/A_{signal})/(N_{norm}/A_{norm})$. So, to the level that the normalization and signal mode topologies are similar, it was expected that differences in acceptance between the Monte Carlo and the data would tend to cancel. Studies were done of the effect of disagreements between the signal mode and normalization mode Monte Carlos by adjusting the Monte Carlos until they agreed and determining the effect on the result.

The percentage systematic errors in the SES due to these sources are shown in Table I. Adding these systematic errors in Table I in quadrature, we obtain a total systematic error in the SES of 7.23%. Taking into account the statistical error of 1.09%, this leads to a total error of 7.31% resulting in a SES error of 5.07×10^{-9}.

The method of Ref [11] was used to obtain an upper limit for $K_L \to \pi^0\pi^0\gamma$. If n_{exp} is the expected number of signal plus background events in the signal box and n_{bkg} is the expected number of background events, then the probability for observing n events is given by a Poisson distribution $P(n_{exp}, n)$ whose mean is

$$n_{exp} = n_{bkg} + BR(K_L \to \pi^0\pi^0\gamma))/SES \qquad (8)$$

A confidence region was constructed containing 90% of the Poisson distribution by varying the $K_L \to \pi^0\pi^0\gamma$ branching ratio between 0 to 5×10^{-7}. Using this confidence region, a 90% CL upper limit for the BR($K_L \to \pi^0\pi^0\gamma$) of 2.43×10^{-7} was determined, assuming the E2 model of Ref, [4]. The SES error was incorporated by varying the SES over the 90% CL range of the SES indicated by its combined statistical and systematic error.

In conclusion, using their 1997 and 1999 data and assuming that the decay proceeds mainly via direct E2 photon emission, the KTeV collaboration obtained an upper limit for the branching ratio for the rare decay $K_L \to \pi^0\pi^0_D\gamma$ of 2.43×10^{-7}, approximately twenty times lower than the best published upper limit [9]. This result provides a much more stringent upper limit for ChPT theoretical calculations of the E2 amplitude.

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